Market Microstructure Invariance: Empirical Hypotheses and *Market Microstructure Invariance: A Dynamic Equilibrium Model* Part 2

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Econ 590

Spring 2020

Working paper recap: Agents

A more detailed market microstructure for examining liquidity invariances.
Agents:

- Informed traders
- Noise traders
- Risk neutral market makers.

Key features:

- informed traders can invest to gather information.
- Adverse selection, but costly information gathering, endogenous participation.

Will compare information gathering decision across assets.
Limitations:

- Informed and noise traders arrive only once in the market.
- Noise traders assumed to endogenously have same trading patterns as information traders (for tractability, modeled as having "fake information")

Asset pricing

Fundamental price follows geometric Brownian motion with volatility σ_F.
Roughly speaking, the market's estimation of the current value $\overline{B}(t)$ of the Brownian motion remains unchanged until someone gathers information.
Without updating the information on the Brownian motion, the fundamental price drifts. Variance of error component $\Sigma(t)$ grows over time.
Thus $(\overline{B}(t), \Sigma(t))$ represents the current state of market information, publicly available.
By paying a fixed amount, an information trader on arrival receives a noisy signal about the true current value of the Brownian motion.

Informed agents

Informed agents' actions (buy or sell) push the price back towards the true value.

- Informed traders arrive at rate $\gamma_I(t)$.
- Details of information process are structured so that informed agents choose trades are in the direction of and proportional to the size of the signal they receive.
- They trade taking account of the effect of their actions on the price.
- Prices assumed to move proportionately to the size of their trades; they choose a degree of trade that causes prices to move by a profit maximizing proportion θ (trading off size of trade versus price effect)

Whoa! Isn't that circular?

If he knows a price increase is associated with a certain level of information, shouldn't the market maker just undo whatever the informed agent does?
Answer: there are noise traders around as well; they increase the size of their sales without respect to the information (this is the assumption that their trades have the same pattern but no information content).
So the market maker assumes that an increase in quantity only has a fraction of the effect because only a fraction of the traders are informed.

Role of informed agents

- If we take the market power model seriously, then it is profit maximizing for the informed agent to make a trade big enough to move the the market makers must believe that there is a fifty-fifty chance that a random trader is informed, and $\theta = 1/2$.
- Notice as well, that the informed trader doesn't use up all his information–the price in the market does not move entirely to the point which would be the informed trader's best guess. This comes from the assumption that each trades only once.
- For future consideration: As precision of information changes, how does trading activity change?

All of these points are addressed in Kyle 85.

Role of uninformed agents

K&O assume each noise trader makes trades each day of total volume in a fraction η of the market.
In the model they endogenously act to make the volumes the same per trade as the informed.
K&O state "the arrival of bets $\gamma(t)$ sets the pace of business time in this model," but it probably ought to be the arrival of informed bets.

Volume

In any event the volume is

$$V(t) = \eta N/(1-\theta)$$

(Thus the volume ultimately depends on the activity of the noise traders η. Think about what this equation means in terms of θ)

Dynamics

Approximately speaking the amount of impact on price of a trade is

$$\theta P(t)\sigma_F E\{\Delta \overline{B}_I(t)\}$$

and this is true whether the trade is from an informed or an uninformed agent. But the informed make money, while the uninformed lose, and the market makers break even.
How does this work? Because of price movements in the future, we can tell if someone was informed or not by the subsequent movements in prices.

Dynamics

But this is a quasi rent for the informed—now we have to include the cost of information gathering—so that profit is dissipated in whatever the information cost is.

At this point, K&O flip to a continuous approximation of the model, rather than assuming jumps for each bet. The result of the calculations is that the underlying information (estimates of $\overline{B}$) filters in over time, as does noise. Price follows a martingale incorporating this, but the volatility of price is stochastic.

This means that $\gamma(t)$ itself depends on the volatility at any moment; in some situations it is more valuable for information gathering to occur.

But is that consistent with θ being determined elsewhere???

Linking the Two Papers

Finally, they argue that when this model is applied under the case of a constant cost and constant value of the signal, that the model generates the invariance assumptions used in the Econometrica paper. Their intuition for the invariance results (page 18) can be summarized as follows:

Linking the Two Papers

Suppose the number of noise traders increases for an exogenous reason by a factor of 4 (their description of the reason is opaque, but I don't think that matters)
As a result, market depth increases (that is, an order moves price less) and the increased profitability increases number of traders, ultimately by a factor of 4 as well.
All variance in price is due to bets by the informed traders. Each still makes a single bet, so each bet only accounts for 1/4 of the returns variance that it did before.
The structural model shows that pricing accuracy and liquidity both increase by a factor of 2, as a result of which informed traders exactly cover the cost of private signals by submitting bets 2 times as large as before.
Overall dollar volume in the market increases by a factor of 8, one-third of the increase in dollar volume comes from changes in bet size and two thirds comes from changes in the number of bets.